LITERATURE CITED

1. P. P. Olodovskii, "Effect of change in disperse-medium density in the solid–adsorbed-water–liquid system. Calculation of the adsorption characteristics of disperse systems," Inzh.-Fiz. Zh., 26, No. 4 (1974).
2. P. P. Olodovskii, "Effect of the surface properties of hydrated solid phase on the change in density of the disperse medium," Inzh.-Fiz. Zh., 32, No. 3 (1977).
3. C. H. Ambery and R. McIntosh, Can. J. Chem., 30 (1952).
4. D. J. C. Yates, Proc. R. Soc., A224 (1954).
5. K. O. Legg and F. P. Jona, J. Phys. C, Solid-State Phys., No. 8, 22 (1975).
6. V. A. Frank-Kamenetskii (editor), Handbook on the X-Ray Investigation of Minerals [in Russian], Nedra, Leningrad (1975).
7. L. A. Gribov, Introduction to Molecular Spectroscopy [in Russian], Nauka, Moscow (1976).
8. L. Bragg and G. F. Claringbull, The Crystal Structure of Minerals, Cornell Univ. Press (1966).

INFLUENCE OF A CENTRIFUGAL FIELD ON THE RESIDUAL PRESSURE IN THE VACUUM CAVITY OF A ROTATING HELIUM CRYOSTAT

I. A. Tsimmerman and A. D. Shvets

UDC 621.5:533.581

The increase in the residual pressure in the vacuum cavity of a rotating helium cryostat subjected to centrifugal forces due to the redistribution of the molecule relative velocities as well as the stripping of molecules being condensed on the cold wall is theoretically computed.

In connection with the development of cryogenic electrical machine construction, it is of great practical interest to study the vacuum thermal insulation of liquid helium under strong centrifugal field conditions.

The vacuum cavity of a rotor cryostat in a typical cryogenic generator [1] is the volume formed by two coaxial cylinders. The outer cylinder is at room temperature, but the inner is at the liquid helium boiling point. It is known that the pressure in a vacuum system at helium temperatures is determined by the partial pressures of such residual gases as helium, hydrogen, and neon. However, in strong centrifugal fields a change occurs in the relative velocities of the molecules moving between the hot and cold walls, as does also the stripping of part of the molecules being condensed (adsorbed) on the cold wall. This can result in degradation of the vacuum in the dynamic mode of cryostat operation, and therefore, in an increase in the heat influx to the liquid helium.

For a quantitative estimate of the influence of the centrifugal field on the residual pressure, let us consider the flux of molecules escaping from the hot to the cold wall and conversely. The molecule thermal energy should evidently be higher than the centrifugal field energy for passage from the hot (outer) to the cold (inner) wall. This means that molecules whose mean thermal velocity $\overline{V}_t$ is greater than the average linear angular velocity of the cryostat V_l will overcome the potential barrier of the centrifugal field. The quantity of such molecules, i.e., the quantity of molecules making an impact per unit surface of the cold wall in unit time, is [2]

$$N_t = n_t V_t / 4. \tag{1}$$

We find the quantity n_t (the molecule concentration with thermal velocity greater than V_l) from the expression [3]

$$\frac{n_t}{n} = \frac{4}{\sqrt{\pi}} \int_{u_0}^{\infty} u^2 \exp(-u^2)\, du, \tag{2}$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 36, No. 3, pp. 533-536, March, 1979. Original article submitted February 13, 1978.

where the relative velocity is $u = V_l / V_t$.

For molecules having a velocity greater than V_l,

$$V_t = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2kT} \right)^{3/2} \int\limits_{V_l}^{\infty} V^3 \exp\left(-\frac{mV^2}{2kT} \right) dV. \tag{3}$$

Solving the integral in this last equation, we obtain a formula to determine the arithmetic mean velocity of the molecules escaping to the cold wall

$$V_t = \frac{2}{\sqrt{\pi}} V_t \left(1 + \frac{V_l^2}{V_t^2} \right) \exp\left(-\frac{V_l^2}{V_t^2} \right). \tag{4}$$

Substituting the values n_t and V_t from (2) and (4) into (1), we obtain

$$N_t = \frac{2}{\sqrt{\pi}} nV_t \left(1 + \frac{V_l^2}{V_t^2} \right) \exp\left(-\frac{V_l^2}{V_t^2} \right) \int\limits_{u_0}^{\infty} u^2 \exp(-u^2)\, du. \tag{5}$$

For the most probable velocities of the molecules entering into the composition of air, the integral in (5) has a value close to unity [4] at a 300°K temperature and a linear angular velocity of the cryostat of 200 ml/sec, viz., 0.9 for N_2 and O_2, and 0.99 for H_2 and He. Consequently, we can write

$$N_t = \frac{2}{\sqrt{\pi}} nV_t \left(1 + \frac{V_l^2}{V_t^2} \right) \exp\left(-\frac{V_l^2}{V_t^2} \right) = \frac{2}{\sqrt{\pi}} \frac{P_t}{kT_t} V_t \left(1 + \frac{V_l^2}{V_t^2} \right) \exp\left(-\frac{V_l^2}{V_t^2} \right). \tag{6}$$

The thermal velocity of molecule motion at T = 300°K is substantially greater than the linear angular velocity of the rotor cryostat of an electrical machine of reasonable size, i.e., $V_t \gg V_l$. The last equation then acquires the more simple form

$$N_t = \frac{2}{\sqrt{\pi}} \frac{P_t}{kT_t} V_t. \tag{7}$$

Let us determine the quantity N_c. In the stationary mode the molecules being condensed (adsorbed) on the cold wall are in a potential well whose depth equals their heat of adsorption Q_a. Molecules whose energy will be greater than the heat of adsorption at a given temperature will be able to leave this well (be desorbed). In conformity with the Boltzmann distribution [3], the number of such molecules is

$$a_c = a_0 \exp\left(-\frac{Q_a}{RT_c} \right). \tag{8}$$

Centrifugal forces in the opposite direction as well as the adsorption forces act on the gas molecules being condensed in a centrifugal field. The depth of the potential well in which the molecules being condensed are at the energetic level RT_c is thereby diminished by the quantity $\mu V_l^2/2$, i.e., by the magnitude of the centrifugal energy of a mole of gas. Therefore, the number of escaping molecules in a centrifugal field is

$$a_c^v = a_0 \exp\left[-\left(\frac{Q_a}{RT_c} - \frac{\mu V_l^2}{2RT_c} \right) \right]. \tag{9}$$

The concentrations of the cold gas in the stationary and dynamic modes will equal, respectively:

$$n_c = a_0 \exp\left(-\frac{Q_a}{RT_c} \right) \delta, \quad n_c^v = a_0 \exp\left[-\left(\frac{Q_a}{RT_c} - \frac{\mu V_l^2}{2RT_c} \right) \right] \delta, \tag{10}$$

where δ is a quantity reciprocal to the volume of the vacuum cavity.

Substituting the value of n_c in the expression for n_c^V, we obtain

$$n_c^v = n_c \exp\left(\frac{\mu V_l^2}{2RT_c} \right). \tag{11}$$

If one-fourth of the molecules escape in a given direction in the stationary mode, then in a strong centrifugal field it can be considered that all the molecules escaping under the effect of the centrifugal force will travel in a radial direction with the velocity $(V_c + V_l)$. Therefore

TABLE 1. Computed Values of the Partial Pressures of Certain Gases in a Centrifugal Field

Gas	Content in air, %	P_c, torr	P_m, torr	P_m/P_c
N_2	78,02	10^{-60}	10^{-53}	10^7
H_2	$1\cdot10^{-2}$	10^{-13}	$2{,}5\cdot10^{-12}$	25
He	$4\cdot10^{-4}$	$5\cdot10^{-10}$	$1\cdot10^{-7}$	200

$$N_c = (V_c + V_l)\, n_c^v = (V_c + V_l)\, n_c \exp\left(\frac{\mu V_l^2}{2RT_c}\right). \tag{12}$$

In the thermodynamic equilibrium state $N_t = N_c$, or

$$\frac{2}{\sqrt{\pi}}\,\frac{P_t}{kT_t}\,V_t = n_c \exp\left(\frac{\mu V_l^2}{2RT_c}\right)(V_c + V_l) = \frac{P_c}{kT_c}(V_c + V_l)\exp\left(\frac{\mu V_l^2}{2RT_c}\right). \tag{13}$$

We find from this latter expression

$$P_t = P_c\,\frac{T_t}{T_c}\,\frac{\sqrt{\pi}}{2}\,\frac{(V_c + V_l)}{V_t}, \tag{14}$$

and

$$P_m = \frac{P_t + P_c}{2} = \frac{1}{2}\,P_c\left[1 + \frac{\sqrt{\pi}}{2}\,\frac{T_t}{T_c}\,\frac{(V_c + V_l)}{V_t}\exp\left(\frac{\mu V_l^2}{2RT_c}\right)\right], \tag{15}$$

which shows the dependence of the residual pressure in the system on the magnitude of the linear angular velocity of the cryostat. For $V_l = 0$ it is transformed into the known Knudsen formula for the thermomolecular effect.

The computed values of the partial pressures P_m of certain gases in a centrifugal field are represented in Table 1 for $V_l = 200$ m/sec, $T_c = 5$°K, $T_0 = 300$°K, and the preliminary rarefaction pressure $P_0 = 1\cdot10^{-4}$ torr.

As is seen from Table 1, although the partial pressures of the individual gases grow tens, hundreds, and thousands of times in the centrifugal field, the total pressure in the system remains sufficiently low. Naturally helium introduces the greatest contribution to the residual pressure. Although the helium pressure in the dynamic mode is indeed 200 times higher than in the stationary mode, it still remains sufficiently low not to exert essential influence on the heat influx to the liquid helium by the residual gas in the cryostat vacuum jacket.

NOTATION

n, concentration of all molecules; V_t, most probable velocity of "thermal" molecules; m, molecule mass; k, Boltzmann constant; T, absolute temperature; P_t, gas pressure at the hot wall; T_t, temperature of the hot wall, N_c, quantity of molecules escaping from unit surface of cold wall per unit time to the hot wall; a_0, number of molecules being condensed on unit cold wall area; R, universal gas constant; μ, gas molecular weight; V_c, most probable velocity of "cold" molecules; P_c, gas pressure at the cold wall which can be considered equal to the saturated vapor pressure at a given temperature; T_c, cold wall temperature; P_m, residual gas pressure in the cryostat vacuum cavity.

LITERATURE CITED

1. V. G. Dan'ko, V. S. Kil'dishev, V. P. Kuchinskii, and L. Ya. Stanislavskii, "Electrical machine with intense cooling," Inventors' Certificate No. 262,240, Byull. Izobret., No. 6 (1970).
2. B. I. Korolev, V. I. Kuznetsov, A. I. Pipko, and V. Ya. Plisovskii, Principles of Vacuum Engineering [in Russian], Énergiya, Moscow (1975).
3. A. K. Kikoin and N. K. Kikoin, Molecular Physics [in Russian], Nauka, Moscow (1976).
4. L. N. Rozanov, Vacuum Machines and Apparatus [in Russian], Mashinostroenie, Moscow (1975).